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ELEMENTARY GEOMETRICAL CONICS.

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SYLLABUS

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ELEMENTARY GEOMETRICAL CONICS.

THE PARABOLA.

Defs. of *parabola*, *focus* (*S*), *directrix*, *symmetry of a curve with respect to a straight line.*

Mechanical tracing.

1. Construction for points on the parabola. The perpendicular on the directrix through the focus is an axis of symmetry.

Defs. of *axis*, *vertex* (*A*), *ordinate*, *abscissa*, *focal distance.*

2. If the chord PP′ intersects the directrix in Z′, SZ′ bisects the exterior angle between SP and SP′.

3. If PN is an ordinate to the parabola at the point P, then $PN^2 = 4AS \cdot AN$.
   Def. of *latus rectum*.
4. The latus rectum $LL' = 4AS$.
   Def. of *tangent*.
5. If the tangent at P meets the directrix in Z, PSZ is a right angle, and the tangent at P bisects the angle between the focal distance SP and the perpendicular PM on the directrix: and the tangent at the vertex is at right angles to the axis.
6. The tangents at the extremities of a focal chord PSP_1 intersect at right angles on the directrix.

Def. of *normal.*

7. If the tangent and normal at P meet the axis in T and G respectively, ST = SP = SG.

Def. of *subtangent.*

8. Subtangent NT = 2AN.

Def. of *subnormal.*

9. Subnormal NG = 2AS.

10. If the tangent at any point P intersects the tangent at the vertex in Y, then SY bisects PT at right angles, and is a mean proportional between SA and SP. ($SY^2 = AS \cdot SP$.)

11. If from any point K on the tangent at P, KI is drawn perpendicular to the directrix, and KU perpendicular to SP, then SU = KI (Adams' property).
12. To draw two tangents to the parabola from an external point O.
13. The two tangents OQ, OQ′ subtend equal angles at the focus, and the triangles SOQ, SQ′O are similar.
14. If a pair of tangents OQ, OQ′ is drawn to a parabola, and OV is drawn parallel to the axis meeting QQ′ in V, QQ′ is bisected in V.

15. The locus of the middle points of any system of parallel chords of a parabola is a straight line parallel to the axis.

Def. of *diameter*: the locus of the middle points of a system of parallel chords.

Def. of *ordinate to a diameter.*

16. If QV is an ordinate to the diameter PV, then $QV^2 = 4SP \cdot PV$.

17. If the focal chord QSQ′ is bisected by the diameter PV, which meets the curve in P, $QQ' = 4SP$.

18. If two chords, QQ′, qq', of a parabola intersect one another, the rectangles contained by their segments are in the ratio of the parallel focal chords;

or $QO \cdot Q'O : qO \cdot q'O = 4SP : 4S\phi$.

ORTHOGONAL PROJECTION, OR REDUCTION OF ORDINATES.

The following propositions may be proved either by orthogonal projection, or by reduction of ordinates in a given ratio.

Definitions.

α. The projection of a straight line on a plane is a straight line, cutting the base line in the same point.

NOTE.—In orthogonal projection the base line is the intersection of the two planes. In reduction of ordinates it is the line to which the ordinates are drawn.

β. The ratio of the segments of a finite straight line is unaltered by projection.

γ. Parallel straight lines project into parallel straight lines of proportional length.

δ. A tangent projects into a tangent, cutting the base line in the same point.

ε. The ratio of areas is unaltered by projection.

ζ. The projections of two straight lines at right angles to one another are, in general, lines at right angles to one another, if one of the original lines is parallel to the base line.

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THE ELLIPSE.

Defs. of *ellipse, focus* (S), *directrix, eccentricity* (e).

1. Construction for points on the ellipse. The perpendicular on the directrix through the focus is an axis of symmetry. To find the vertices A, A.′ The curve lies between lines drawn through A, A′ at right angles to the axis.
2. If the chord PP′ intersects the directrix in Z′, SZ′ bisects the exterior angle between SP and SP′.

Defs. of *major axis* (AA'), *centre* (C), *minor axis* (BB').

3. If PN is the ordinate of a point P on the ellipse, $PN^2 : AN \cdot A'N = CB^2 : CA^2$, and CB is less than CA.

4. If the ordinates of the circle described on AA′ as diameter be reduced in the ratio of CB : CA, the locus of their extremities is the ellipse ($PN : pN = CB : CA$).

Def. of *auxiliary circle.*

5. The projection of a circle is an ellipse.

NOTE.—This Prop. and Prop. 28 may be proved either by orthogonal projection, or by reduction of ordinates.

6. The ellipse is symmetrical with respect to the minor axis, and has a second focus (S′) and directrix.

Also all chords passing through C are bisected at C.

7. $CA = e \cdot CX$; $CS = e \cdot CA$; $CS \cdot CX = CA^2$.
8. $SP + S'P = AA'$. Mechanical construction for ellipse.
9. $CB^2 = CA^2 - CS^2 = SA \cdot SA'$.
   Def. of *latus rectum* (LL').
10. $SL \cdot CA = CB^2$.

11. If the tangent at P meets the directrix in Z, PSZ is a right angle.

    Also tangents at the ends of a focal chord intersect on the directrix.

12. If the normal at P intersects the major axis in G, $SG = e \cdot SP$.

13. The tangent and normal to an ellipse at any point P are respectively the external and internal bisectors of the angle between the focal distances.

14. The foot of the perpendicular (SY) from the focus on the tangent at P is on the auxiliary circle.

    Also if CE, parallel to the tangent, intersects S'P in E, $PE = CA$.

    Also $SY \cdot S'Y' = CB^2$.

Defs. of *corresponding points of ellipse and auxiliary circle, corresponding chords.*

15. Corresponding chords of the ellipse and auxiliary circle intersect on the major axis.

Also tangents at corresponding points intersect on the major axis.

16. If the tangent at P meets the major axis produced in T, $CN \cdot CT = CA^2$.

17. If the tangent at P meets the minor axis produced in t, and Pn is the perpendicular from P on the minor axis, $Cn \cdot Ct = CB^2$.

18. If PF is the perpendicular from P on a line through C parallel to the tangent at P, and if the normal at P meets the minor axis in g, then

$$PF \cdot PG = CB^2, \text{ and } PF \cdot Pg = CA^2.$$

19. $NG : CN = CB^2 : CA^2$; and $CG = e^2 \cdot CN$.

20. If from any point K on the tangent at P, KI is drawn perpendicular to the directrix, and KU perpendicular to SP, then $SU = e.KI$ (Adams' property).
21. To draw a pair of tangents OQ, OQ′ to an ellipse from an external point O.
22. Tangents OQ, OQ′ subtend equal angles OSQ, OSQ′ at the focus S.
23. Tangents OQ, OQ′ are inclined at equal angles to OS, OS′.

24. The locus of the middle points of any system of parallel chords of an ellipse is a straight line passing through the centre ; and the tangent at either end of the straight line is parallel to the chords.

Def. of *diameter* as the locus of the middle points of a system of parallel chords.

NOTE.—The word diameter is frequently used to denote the length of the portion of this straight line intercepted by the curve.

Def. of *ordinate to a diameter.*

25. Tangents at the ends of any chord meet on the diameter which bisects that chord.

26. QV is an ordinate of the diameter CP ; if the tangent at Q meets CP produced in O, then $CV \cdot CO = CP^2$.

27. If CP bisects chords parallel to CD, then CD bisects chords parallel to CP.

Def. If two diameters are so related that each bisects chords parallel to the other, they are said to be *conjugate.*

28. Conjugate diameters in the ellipse are the projections of diameters in the circle at right angles to one another.

Def. of *supplemental* chords.

29. Supplemental chords are parallel to conjugate diameters.

30. QV is an ordinate of the diameter PCP′, CD the diameter parallel to QV, then

$$QV^2 : PV \cdot P'V = CD^2 : CP^2.$$

31. If DR be the ordinate of D,

$$CR : PN = CA : CB, \text{ and } CN : DR = CA : CB.$$

32. $CP^2 + CD^2 = CA^2 + CB^2$.

33. $PF \cdot CD = CA \cdot CB$.

The area of the parallelogram formed by tangents at the extremities of conjugate diameters is constant.

34. If two chords of an ellipse intersect, the rectangles contained by their segments are as the squares of the semi-diameters.

THE HYPERBOLA.

Def. of *hyperbola*, *focus* (*S*), *directrix*, *eccentricity* (*e*).

1. Construction for points on the hyperbola. The perpendicular on the directrix through the focus is an axis of symmetry. To find the vertices A, A′. The curve lies outside lines drawn through A, A′ at right angles to the axis.
2. If the chord PP′ intersects the directrix in Z′, SZ′ bisects the exterior angle between SP and SP′.

3. If PN is an ordinate of a point P on the curve, $PN^2 : AN \cdot A'N$ is a constant ratio.

Defs. of transverse axis, centre, conjugate axis.

$$PN^2 : AN \cdot A'N = CB^2 : CA^2.$$

4. If the diagonals of the rectangle, formed by perpendiculars through the extremities of the axes ACA', BCB', be produced indefinitely, and the ordinate NP be produced both ways to meet them in p, p', the rectangle $Pp \cdot Pp' = CB^2$.

Also the curve continually approaches to each diagonal without actually meeting it, and its distance from it becomes ultimately less than any finite length.

Def. of Asymptote. When a curve continually approaches to a fixed straight line without ever actually meeting it, but so that its distance from it becomes ultimately less than any finite length, the line is said to be a rectilinear asymptote to the curve.

5. The hyperbola is symmetrical with respect to the conjugate axis, and has a second focus and directrix.

Also all chords passing through C are bisected at C.

6. $CA = e \cdot CX$; $CS = e \cdot CA$; $CS \cdot CX = CA^2$.
7. $SP \sim S'P = AA'$. Mechanical construction for hyperbola.
8. $CB^2 = CS^2 - CA^2 = SA \cdot SA'$.
   Def. of *latus rectum* (LL').
9. $SL \cdot CA = CB^2$.

10. If the tangent at P meets the directrix in Z, PSZ is a right angle, and tangents at the ends of a focal chord intersect on the directrix.

11. If the normal at P intersects the transverse axis in G, $SG = e \cdot SP$.

12. The tangent and normal to a hyperbola at any point P are respectively the internal and external bisectors of the angle between the focal distances.

13. The foot of the perpendicular SY on the tangent at P, lies on the circle described on AA′ as diameter.

Also if CE, parallel to the tangent, intersects S′P in E, $PE = CA$. Also $SY \cdot S'Y' = CB^2$.

14. If the tangent at P meets the transverse axis produced in T, $CN \cdot CT = CA^2$.

15. If the tangent at P meets the conjugate axis produced in t, and Pn is the perpendicular from P on the conjugate axis, $Cn \cdot Ct = CB^2$.

16. If PF is the perpendicular from P on a line through C parallel to the tangent at P, and if the normal at P meets the conjugate axis in g, then

$$PF \cdot PG = CB^2, \text{ and } PF \cdot Pg = CA^2.$$

17. $NG : CN = CB^2 : CA^2$; and $CG = e^2 \cdot CN$.

18. If from any point K on the tangent at P, KI is drawn perpendicular to the directrix, and KU perpendicular to SP, then $SU = e \cdot KI$ (Adams' property).
19. To draw a pair of tangents OQ, OQ′ to an hyperbola from an external point O.
20. Tangents OQ, OQ′ subtend equal or supplementary angles OSQ, OSQ′ at the focus S according as Q, Q′ are on the same or opposite branches of the hyperbola.
21. Tangents OQ, OQ′ are inclined at equal or supplementary angles to OS, OS′ according as Q, Q′ are on opposite or the same branches of the hyperbola.

Def. An hyperbola which has CB and CA for transverse and conjugate axes respectively is called the *conjugate hyperbola.*

NOTE.—The conjugate hyperbola has the same asymptotes as the original hyperbola.

22. If through any point P on the hyperbola or its conjugate a line be drawn parallel to CA or CB, meeting the asymptotes in p, p', the rectangle Pp. Pp' is equal to the square on CA or CB respectively.

23. If through any two points P, Q, on the hyperbola or its conjugate two parallel straight lines be drawn to meet the asymptotes in p, p', q, q', respectively, the rectangle Pp. $Pp' = Qq$. Qq'.

24. If any straight line cuts the curves in Q, Q', and the asymptotes in q, q', $Qq = Q'q'$.

Also if the tangent at P meets the asymptotes in r and r', $Pr = Pr'$.

25. The locus of the middle points of a system of parallel chords of an hyperbola is a straight line passing through the centre; and the tangents at the points where the straight line meets the curve are parallel to the chords.

Def. of *diameter* as the locus of the middle points of a system of parallel chords.

Def. of *ordinate to a diameter.*

NOTE.—If a series of parallel lines forms chords of the hyperbola and its conjugate, the same diameter bisects both systems of chords.

26. If one diameter bisects chords parallel to a second, then the second diameter bisects chords parallel to the first.

Def. If two diameters are so related that each bisects chords parallel to the other, they are called *conjugate diameters.*

NOTE.—Of two conjugate diameters one will meet the hyperbola, and the other the conjugate hyperbola.

Def. *Supplemental chords.*

27. Supplemental chords are parallel to conjugate diameters.

28. Tangents at the extremities of conjugate diameters PCP', DCD' form a parallelogram whose angular points are on the asymptotes.

Also PD is bisected by one asymptote and is parallel to the other.

29. Straight lines through P and D parallel to the axes form a rectangle with two angular points on one of the asymptotes.

30. $CP^2 \sim CD^2 = CA^2 \sim CB^2$.

31. If a tangent to the hyperbola meets the asymptotes in r and r' the area rCr' is constant.

Also the area $CPrD$ is constant;

or $PF \cdot CD = AC \cdot BC$.

32. QV is an ordinate of the diameter PCP′, CD the diameter parallel to QV, then

$$QV^2 : PV \cdot P'V = CD^2 : CP^2.$$

33. Tangents at the extremities of any chord meet on the diameter which bisects that chord.

34. QV is an ordinate of the diameter CP : if the tangent at Q meets CP produced in O, then $CV \cdot CO = CP^2$.

35. If two chords of an hyperbola intersect, the rectangles contained by their segments are as the squares of the parallel semi-diameters.

CYLINDER AND CONE.

Def. *Right circular cylinder.*

1. The section of a right circular cylinder by a plane inclined to the axis is an ellipse.

Defs. of *right circular cone, its axis, its vertex, focal sphere, axial plane.*

NOTE.—The section of a cone by a plane passing through the vertex is either a point, or two generating lines of the cone.

The section of a cone by a plane, perpendicular to the axis, not through the vertex, is a circle.

Def. When a cone is cut by a plane, the plane passing through the axis of the cone and perpendicular to the cutting plane is called the *axial plane.*

NOTE.—The intersection of the axial plane with the cutting plane is an axis of the curve of section; and its intersection with the cone is two generating lines.

2. The section of a cone by a plane, not passing through the vertex and not perpendicular to the axis, satisfies the definition of a conic section (SP = e . PM).

   The focus is the point of contact of the cutting plane with the focal sphere, and the directrix is the intersection of the cutting plane with the plane containing the circle of contact of the focal sphere with the cone.

3. A plane section of a cone is an ellipse if its focal axis meets both generating lines in the axial plane on the same sheet of the cone; it is a parabola if its axis is parallel to one of the generating lines; it is an hyperbola if its focal axis meets both generating lines, but on different sheets of the cone.

4. In an elliptic section of a cone, the major axis is equal to the distance between the focal spheres measured along a generating line of the cone.
5. In an hyperbolic section of a cone, the transverse axis is equal to the distance between the focal spheres measured along a generating line of the cone.

6. In a parabolic section of a cone, the latus rectum is a third proportional to the distance of the vertex of the cone from the vertex of the parabola and the diameter of the circular section of the cone through the vertex of the parabola.
7. In an elliptic section of a cone, the minor axis is a mean proportional between the diameters of the circular sections of the cone passing through the ends of the major axis.

8. In an hyperbolic section of a cone, the conjugate axis is a mean proportional between the diameters of the circular sections of the cone passing through the vertices of the hyperbola.
9. The asymptotes of an hyperbolic section of a cone are parallel to the two generating lines, which lie in a parallel plane through the vertex of the cone.

10. If through any point two straight lines are drawn, parallel to two fixed straight lines, to intersect a given cone, the ratio of the rectangles contained by the segments of the lines is constant for all positions of the point.

HERBERT M. COHEN
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